NASA TECHNICAL TRANSLATION NASA TT F-13,276

# OPERATIONAL AIR TRAFFIC CONTROL CONSIDERING SEVERAL OPTIMALITY CRITERIA

A. I. Volevach, V. L. Volkovich, L. F. Dargeyko

Translation of "Operativnoye upravleniye vozdushnym dvizheniyem s uchetom neskol'kikh kriteriyev optimal'nosti," IN: Slozhnye sistemy upravleniya [Complex Control Systems], Naukova Dumka Press, Kiev, 1969, pp. 36-43.



NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546 OCTOBER 1970

NASA TT F-13,276

# OPERATIONAL AIR TRAFFIC CONTROL CONSIDERING SEVERAL OPTIMALITY CRITERIA

A. I. Volevach, V. L. Volkovich, L. F. Dargeyko

ABSTRACT. Development of a two-level system of air traffic control which avoids conflicts between aircraft on the runway and in flight, and which minimized deviations from the prescribed landing times. The problem of avoiding conflict between aircraft during flight is reduced to making a complex decision with allowance for several optimality criteria and for a limited number of control inputs. An algorithm for selecting the control parameter on the basis of compromises is proposed. A70-25603

## Selection of optimality criteria and statement of the problem /36*

An automatic operational flight control system for a set of aircraft in some control zone should provide the solutions to the following basic problems. The first of these involves the determination of the sequence and prescribed landing or takeoff times for the aircraft, with the requirement that no conflicts occur on the runway taken into consideration. The second involves conflict-free traffic on the airways. The elimination of conflict means the exercise of control such that there will be no future conflicts with other aircraft, that deviations from prescribed landing times, or from planned zone departure times, will be minimized, and that control costs will be minimized.

These criteria are governed by the following objective functions.

The number of conflict situations - by the number of unsatisfied inequalities:

(1) for aircraft flying on parallel courses

$$[(t_i^\alpha - t_i^\beta)(t_{i+1}^\alpha - t_{i+1}^\beta) < 0] \wedge [(h_0^\alpha - h_0^\beta) \geqslant \Delta h], \tag{1}$$

where

$h_o^\alpha$, $h_o^\beta$, are the altitudes of the overtaking points $\alpha$ and $\beta$ for the aircraft;

---

* Numbers in the margin indicate pagination in the foreign text.

$t_i^\alpha$, $t_{i+1}^\alpha$ are the flight times for the $\alpha^{th}$ aircraft for the beginning and end of the common section of the track on which the overtaking, or the rendezvous, will occur;

$\Delta h$ is the minimum permissible altitude between the aircraft;

(2) for aircraft flying on intersecting courses

$$[|h_l^{\gamma+1} - h_l^{\gamma}| \geqslant \Delta h] \wedge [(t_l^{\gamma+1} - t_l^{\gamma}) < \tau] \vee \vee [(t_l^{\gamma+1} - t_l^{\gamma}) > \tau] \wedge [|h_l^{\gamma+1} - h_l^{\gamma}| < \Delta h], \tag{2}$$

where

$\tau$ is the minimally permissible time interval for aircraft closing;

$t_l^\gamma$ is the time the aircraft with priority flies through the $l^{th}$ point.

The criterion that takes into consideration deviation from prescribed landing or zone departure time is determined by the objective function

$$f_2^\beta(u) = |t^\beta(u) - t^\beta|, \tag{3}$$

where

$t^\beta$ is the prescribed landing time for landing aircraft, or the time of departure from the control zone for aircraft that are taking off, or flying through. /37

The criterion that takes control cost into consideration is in the form

$$f_c^\beta = c_j^\beta u_j^\beta \quad (u_j^\beta \in U), \tag{4}$$

where

$u_j^\beta$ is the $j^{th}$ form of the controlling parameter for the $\beta^{th}$ aircraft;

$c_j^\beta$ is the weight of the $j^{th}$ controlling parameter;

$U$ is a set of controlling parameters.

As has been shown in [2], a system for the automatic operational control of the flights of a set of aircraft in a control zone is a system with a clearly defined, closed hierarchical structure with two hierarchical levels. The first level consists of the local subsystems, the individual aircraft with no direct links with each other. Each of the local subsystems (individual aircraft) has optimality criteria, the objective functions of which are determined by Eqs. (1) - (4). The local subsystems are combined by introducing global optimality criteria for the second hierarchical level (airdrome control zone). The global criteria for the second hierarchical level are determined by the objective functions, F, and these depend on the sum of the individual objective functions for the subsystems.

Verification of the inequalities at (1) and (2) for the entire set of aircraft in a zone will determine the total number of conflict situations. The second level problem is one of controlling the flights of the aircraft in the zone so as to minimize the total number of conflict situations, and this is the optimality criterion for the second level.

The second criterion for the second level, and one that gives consideration to the deviation from prescribed times for landing, or for departing the control zone, can be established by the objective function

$$F_2 = \sum_{\alpha} | t^{\alpha}(u) - t^{\alpha} |. \tag{5}$$

Finally, the third second level criterion, the one that takes control cost into consideration, can be established through the expression

$$F_3 = \sum_{\alpha} c_j^{\alpha} u_j^{\alpha} \qquad (u_j^{\alpha} \in U). \tag{6}$$

The sums for these criteria are taken for the entire set of aircraft making up the conflict group.

Thus, the presence of several criteria for some set of types of controlling parameters establishes the problem of air traffic control as a problem of arriving at a complex decision for objects with a hierarchical structure [1].

Preference is given to the importance of $F_1 > F_2$ and $F_3$ for the first criterion, with $F_2$ anf $F_3$ considered as being of equal value. $F_1$ must attain the absolute optimum, that is, the number of conflicts should equal zero, for the criterion. Since control can be exercised by different types of controlling parameters when there is a conflict, the solution to the problem in terms of the first criterion will be found for some set of values for the controlling parameters. Then we find one of the set of controlling parameters that best corresponds to the remaining two criteria, $F_2$ and $F_3$; that is, we find a compromise controlling action. /38

Let us consider the problem of automatically controlling the flights of a set of aircraft in some control zone. For simplicity in what is to follow we will consider only the arriving aircraft, because the control algorithm for those taking off is similar.

Let there be N-1 aircraft in a zone. Let us designate this set of aircraft as $S = \{s_\alpha^j\}$ $(\alpha, j \in I = \{1, \ldots, N-1\})$. The superscript indicates the sequence in which the aircraft will appear in the control zone, and the subscript the landing sequence. There is, in the zone indicated, a network of tracks, the positions of which will not change with respect to time, and which is fixed by an ordered set of control points

$$M^p = \{m_0, m_1, \ldots, m_i^p, \ldots, m_k^p\}, \quad p = 1, 2, \ldots, P,$$

where the superscript, p, is the track number, and the subscript is the number of the control point. The index 0 corresponds to the runway, 1 to the glide path point, and k to the control point for the $p^{th}$ track. We shall designate the groups of aircraft flying one track by $S_p$ (where p indicates the group belongs to the $p^{th}$ track) and $\bigcup_{p=1}^{P} S_p = S$. We will take it that the sequence, and the prescribed landing times have been established for all the aircraft of set S. This will enable us to establish the precribed time and altitudes for the control points along track $M^p$ for the aircraft in set $S_p$

$$T^\alpha = \{t_0^\alpha, \ldots, t_i^\alpha, \ldots, t_k^\alpha\},$$

$$H^\alpha = \{0, \ldots, h_i^\alpha, \ldots, h_k^\alpha\},$$

where $t_i^\alpha$ is the prescribed flight time for aircraft $S_\alpha^i$ with scheduled landing $\alpha m_i^{th}$ for a point on the $p^{th}$ track. The sense of $h_i^\alpha$ is similar. Let us assume that the above indicated problems have been solved for the aircraft in set S. When the $N^{th}$ aircraft appears its turn, $\beta$, and prescribed landing time can be established (the delay $\Delta t^\beta$ in order to eliminate conflict on the runway can be established). Knowing the track the aircraft is flying, that is, the subset, /39
$S_p$, to which it belongs, we can compute the prescribed time and the altitude for the control points along the $p^{th}$ track

$$T^\beta = \{t_0^\beta, \ldots, t_i^\beta, \ldots, t_k^\beta\},$$

$$H^\beta = \{0, \ldots, h_i^\beta, \ldots, h_k^\beta\}.$$

Since conflict on the runway has been eliminated, what remains is to provide for conflict-free flight along the track. When aircraft are making their landings in a specified control zone, the only time conflict possible is when overtaking.

Overtaking will occur if there are aircraft of set $S_p$ landing after the $\beta^{th}$ aircraft. Let us designate the set of these aircraft as $S_p^o$. The overtaking sections can be found by verifying the inequality

$$(t_i^\alpha - t_i^\beta)(t_{i+1}^\alpha - t_{i+1}^\beta) < 0 \tag{7}$$

for each of the control points along track $M^p$ and for each of the aircraft of set $S_p^o$. If the inequality is not satisfied, overtaking will occur on section [i, i + 1], and we find the overtaking point of the aircraft with landing sequence $\alpha$ by the aircraft under consideration. We verify that conflict-free traffic will exist at each of these points

$$|h_i^\alpha - h_i^\beta| \geqslant \Delta h. \tag{8}$$

If Eq. (8) is not satisfied, conflict at the point is possible.

The controlling actions applicable during overtaking can be of two types: (1) a lateral maneuver at some angle $\varkappa$ and return to the flight track; and (2) change in the overtaking point altitude. Let us designate this set of controlling actions by $U = \{u_1, u_2\}$. Depending on which of the aircraft is controlled, each type of control comprises several controlling parameters: $u_i = \{u_i^\alpha, u_i^\beta, u_i^{\alpha\beta}\}$, $i = 1, 2$. The superscript indicates which of the aircraft is being controlled in order to do away with the conflict.

Yet another type of controlling action, in addition to the above two, can be used to eliminate conflict at track intersection points, that of change in air speed. Let us designate this type by $u_3 = \{u_3^\gamma, u_3^{\gamma+1}, u_3^{(1)}, u_3^{(2)}\}$. Change in air speed is equivalent to changing the time the aircraft flies through the point of intersection. Therefore, time delays at a specified point will ensure conflict-free movement. Let us determine these delays. Let the $N^{th}$ aircraft fly through the $l^{th}$ point of intersection with sequence $\gamma$, and let the $j^{th}$ aircraft, the conflicting aircraft, fly through this same point with sequence $\gamma + 1$.

Conflict with some one of the aircraft in the set $S_p$ can occur when the $N^{th}$ aircraft is controlled. Let us combine all potentially conflicting aircraft into a group $S'_p$, that contains the aircraft satisfying the inequality /40

$$t_l^{\gamma-k} - t_l^{\gamma-k-1} \leqslant 2\tau, \quad k = 0, 1, \ldots \tag{9}$$

Let group $S'_p$ contain the aircraft with sequence in flying through the $l^{th}$ point $I = \{\gamma - k_1, \ldots, \gamma - 1, \gamma\}$, where $k_1$ is a number that will not satisfy the inequality of Eq. (9). The time delay, $\Delta t^{\gamma}$ required to eliminate conflict at the $l^{th}$ point when the $N^{th}$ aircraft is the one controlled can be determined as

$$\Delta t^{\gamma} = \tau - t_l^{\gamma+1} + t_l^{\gamma}. \tag{10}$$

Control is exercised on the section preceding the specified point. But since the $N^{th}$ aircraft can conflict with the group of aircraft designated as $S'_p$, the time delays for the aircraft of set $S'_p$ required to eliminate these conflicts can be found as follows

$$\begin{gathered} \Delta t^{\gamma-k-1} = \tau + t_l^{\gamma-k-1} - t_l^{\gamma-k} + \Delta t^{\gamma-k}, \text{ if } t_l^{\gamma-k} + \Delta t^{\gamma-k} - t_l^{\gamma-k-1} < \tau, \\ \Delta t^{\gamma-k-1} = 0, \text{ if } t_l^{\gamma-k} + \Delta t^{\gamma-k} - t_l^{\gamma-k-1} \geqslant \tau, \\ k = 0, 1, 2, \ldots, k_1. \end{gathered} \tag{11}$$

The set of time delays for the entire set of aircraft contained in $S_p$

$$\Delta T_1 = \{0, 0, \ldots, 0, -\Delta t^{\gamma-k_1}, \ldots, -\Delta t^{\gamma}, 0, \ldots, 0\}$$

will fix the magnitude of the controlling action, $u_3^{\gamma}$. The minus sign means that the delay is carried out by increasing the speed at which the aircraft fly the preceding section of the track.

The following inequality will be applicable for the aircraft in a potentially conflicting group when the $j^{th}$ aircraft is controlled

$$t_l^{\gamma+i+2} - t_l^{\gamma+i+1} < 2\tau, \quad i = 0, 1, \ldots \tag{12}$$

The time delays for this group of airplanes have positive signs, corresponding to the reduction in the speed at which they fly over the preceding section of the track, and they can be found for the $\gamma + 1$ aircraft by Eq. (10), as well as for the remaining aircraft of the set, $S'_{p_1}$, as follows

$$\begin{gathered} \Delta t^{\gamma+2+i} = \tau + t_l^{\gamma+1+i} + \Delta t^{\gamma+1+i} - t_l^{\gamma+2+i}, \\ \text{if } \quad t_l^{\gamma+2+1} - t_l^{\gamma+1+i} - \Delta t^{\gamma+1+i} < \tau, \\ \Delta t^{\gamma+2+i} = 0, \text{ if } t_2^{\gamma+2+i} - t_l^{\gamma+1+i} - \Delta t^{\gamma+1+i} \geqslant \tau, \end{gathered} \tag{13}$$

where $i = 0, 1, \ldots, i_1$ [$i_1$ is a number that will not satisfy the inequality of /41

Eq. (12)]. The set of delays for the whole of group $S'_{p_1}$

$$\Delta T_2 = \{0, \ldots, 0, \Delta t^{\gamma+1}, \ldots, \Delta t^{\gamma+1+l_1}, 0, \ldots, 0\}$$

will determine the magnitude of the controlling action $u_3^{\gamma+1}$.

Two variants of the selection of the controlling actions, $u_3^{(1)}$ and $u_3^{(2)}$ are possible if control is exercised over both conflicting aircraft. In the case of $u_3^{(1)}$ the aircraft with sequence $\gamma$ is moved forward by the magnitude

$$\Delta t^{\gamma,(1)} = \tau + t_l^{\gamma-1} - t_l^{\gamma}, \tag{14}$$

while the aircraft with sequence $\gamma + 1$ is moved back by the magnitude $\Delta t^{\gamma+1}$, where

$$\Delta t^{\gamma+1} = \Delta t^{\gamma} - \Delta t^{\gamma,(1)}, \tag{15}$$

and $\Delta t^{\gamma}$ can be found through Eq. (10). The rest of the aircraft in the $S'_{p_1}$ group are moved back, and Eq. (13) provides the time delays for them, with the result that the set of delays for $u^{(31)}$ is in the form

$$\Delta T_3^{(1)} = \{0, \ldots, 0 - \Delta t^{\gamma,(1)}, \Delta t^{\gamma+1,(1)}, \ldots, \Delta t^{\gamma+1+l_2,(1)}, 0, \ldots, 0\}.$$

When $u_3^{(2)}$, the aircraft with landing sequence $\gamma + 1$ will be moved back by the magnitude $\Delta t^{\gamma+1,(2)} = \tau + t_l^{\gamma+1} - t_l^{\gamma+2}$, and the $\gamma^{\text{th}}$ will be moved forward by the magnitude $\Delta t^{\gamma,(2)} = \Delta t^{\gamma} - \Delta t^{\gamma(2)}$. The aircraft in the $S'_p$ set will be moved by the magnitudes of the time delays found through Eq. (11).

Then $$\Delta T_3^{(2)} = \{0, \ldots, 0, -\Delta t^{\gamma-k_2,(2)}, \ldots, -\Delta t^{\gamma,(2)}, \Delta t^{\gamma+1,(2)}, 0, \ldots, 0\}$$

is the set of delays for $u_3^{(2)}$.

Thus, we obtain the following set of controlling actions for eliminating the conflict at the overtaking point

$$U_o = \{u_1 = (u_1^{\alpha}, u_1^{\beta}, u_1^{\alpha\beta}), \quad u_2 = (u_2^{\alpha}, u_2^{\beta}, u_2^{\alpha\beta})\},$$

and the following set for eliminating the conflicts at the point where tracks intersect

$$U_{int} = \{u_1 = (u_1^{\gamma}, u_1^{\gamma+1}, u_1^{\gamma,\gamma+1}), \quad u_2 = (u_2^{\gamma}, u_2^{\gamma+1}, u_2^{\gamma,\gamma+1}), \\ u_3 = (u_3^{\gamma}, u_3^{\gamma+1}, u_3^{(1)}, u_3^{(2)})\},$$

and the conflict control problem will involve the selection of one of the above-indicated controlling parameters best satisfying criteria $F_2$ and $F_3$.

An algorithm for selecting the compromise controlling parameter /42

Since each type of controlling action consists of several controls, the selection of the best of them for each type is the first thing to be done. Two criteria are used to make this selection, so this particular problem is one of arriving at a complex solution. The algorithm for solving the problem comprises the following [3]. Let us find the values of the optimality criteria for each of the controlling actions of type $u_i$

$$F_j^k = F_j(u^k), \quad k = \alpha, \beta, \alpha\beta; \; j = 2, 3.$$

Let us set the magnitude of the least value of $F_j^k$ equal to one, and that of the greatest value of it equal to zero, or

$$\min\{F_j^\alpha, F_j^\beta, F_j^{\alpha\beta}\} = 1,$$
$$\max\{F_j^\alpha, F_j^\beta, F_j^{\alpha\beta}\} = 0, \quad j = 2, 3.$$

All the other magnitudes of the values for the criteria, given the unit scale selected, will equal some proper fraction, and the loss matrix will have the form

$$\begin{array}{c} \\ u_i^\alpha \\ \cdot \\ \cdot \\ \cdot \\ u_i^{\alpha\beta} \end{array} \begin{array}{c} \begin{array}{cc} F_2 & F_3 \end{array} \\ \left| \begin{array}{cc} \dfrac{a_2^\alpha}{b_2^\alpha} & \cdot \\ \cdot & \cdot \\ \cdot & \dfrac{a_3^{\alpha\beta}}{b_3^{\alpha\beta}} \\ \cdot & \end{array} \right| \end{array}, \qquad (16)$$

where $a_j^{(k)} > b_j^{(k)}$ for all k and j, and the zeros and ones occupy the positions of minimal and maximal losses, respectively. The objective function in the following form can serve as the global criterion in this case

$$W = \sum_{j=2}^{3} \frac{a_j^{(k)}}{b_j^{(k)}} \qquad (k = \alpha, \beta, \alpha\beta) \qquad (17)$$

and the problem of selecting the compromise solution involves finding that $u_j^{(k)}$ with the highest value in the linear form of Eq. (17). An analogous algorithm can be used when there are four controlling actions within the type.

Finding the compromise controlling action for each of the types in set

$U_o$, or $U_{int}$, we select the best one. The algorithm for finding the best type of controlling action to eliminate conflict at the point where tracks intersect is as cited above. The algorithm for finding the best type to eliminate conflict /43 during overtaking is as follows. Let us introduce a magnitude characterizing the relative deviation for each of the criteria with respect to the optimal value for each

$$w(u_i, F_j) = \frac{F_j(u_i)}{F_j^{pref}} \qquad (i = 1, 2;\ j = 1, 2)$$

(because the optimal values for the criteria equal zero), where $F_j^{pref}$ has the dimensionality of the j^th^ criterion and the identical magnitude for all criteria. Let us compose a matrix of relative deviations

$$\begin{array}{ccc} & F_2 & F_3 \\ \begin{array}{c} u_1 \\ u_2 \end{array} & \multicolumn{2}{c}{\begin{pmatrix} w(u_1, F_2) & w(u_1, F_3) \\ w(u_2, F_2) & w(u_2, F_3) \end{pmatrix}}, \end{array} \qquad (18)$$

in which $w(u_i, F_j)$ is a number characterizing the preference of the criterian $F_j$ for controlling action $u_i$, as compared with the other terms in the column. Since criteria $F_2$ and $F_3$ are of equal value, the compromise controlling action will be the one for which

$$W(u_i) = w(u_i, F_2) + w(u_i, F_3),\ i = 1, 2 \qquad (19)$$

assumes the least value.

Use of the compromise controlling parameter found will change the landing time, or the time the aircraft flies out of the zone, and this will result in a change in the time the control points are overflown. Violation of the flight plan in this case requires a recomputation of the prescribed landing time, or the prescribed time of departure from the zone. In order to avoid this, it is necessary to use combination controlling actions (change in vertical and horizontal speeds, and change in speed over other sections of the track), that is, to ensure the absolute minimum with respect to the optimality criterion, $F_2$. Moreover, it must be pointed out that control will have to begin with the appearance of the first aircraft in the control zone in accordance with the above-indicated algorithm.

## REFERENCES

1. Vilkas, E.P., Mayminas, Ye.Z. Trudy I Vsesoyuznoy konferentsii po ekonomicheskoy kibernetike [Proceedings of the Ist All-Union Conference on Economic Cybernetics], Batumi, 1966.

2. Kukhtenko, A.I., IN: Slozhnyye sistemy upravleniya [Complex Control Systems]. "Naukova dumka" Press, Kiev, 1966.

3. Lewis, R.D., Raifa, H. Igry i resheniya [Games and Decisions], Foreign Language Press, Moscow, 1961.

Translated for the National Aeronautics and Space Administration under contract No. NASw-2038 by Translation Consultants, Ltd., 944 S. Wakefield St., Arlington, Virginia 22204.